

## 上海交通大学张晓东教授在我院做了学术报告

应我院邀请，上海交通大学数学科学学院张晓东教授于 2022 年 3 月 17 日北京时间 10 点到 11 点通过腾讯会议给我院师生作了一场学术报告。我院青年教师、研究生等共 40 多人听了报告。

张晓东教授的报告的题目是《The signless Laplacian spectral radius of graphs without intersecting odd cycles》，该报告主要是关于不包含图  $H$  作为子图的图  $G$  的无符号拉普拉斯谱半径方面的研究工作，其中图  $H$  是  $k$  个奇长圈恰好在一个共同的顶点上相交的图。在本次讲座中张晓东教授从三个方面进行报告，首先介绍了基础知识和相关的背景以及其他学者对该类图研究的相关结果，其次给出了不包含图  $H$  作为子图的图  $G$  的无符号拉普拉斯谱半径的一个上界，并刻画了得到该上界的所有极图，最后对主要定理进行了简单的证明。报告后，几位青年教师问了几个问题，张晓东教授作了很详细的解答，并且大家进行了激烈的讨论与交流。



### The signless Laplacian spectral radius of graphs without intersecting odd cycles

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This talk is based on work joined with Ming-Zhu Chen (陈明珠, 海南大学),

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## Spectral version for $F_{k,r}$ -friendship graphs

### Theorem 15 (Desai-Kang-Li-Ni-Tait, 2021+)

Let  $G$  be a graph of order  $n$  that does not contain a copy of  $F_{k,r}$ , where  $k \geq 1$  and  $r \geq 2$ . For sufficiently large  $n$ , if  $G$  has the maximal spectral radius, then  $G \in Ex(n, F_{k,r})$ .

### Conjecture 16 (Desai-Kang-Li-Ni-Tait, 2021+)

For integers  $k \geq 1$  and  $r \geq 3$ , there exists an integer  $n_0(k, r)$  such that if  $n \geq n_0(k, r)$  and  $G$  is an  $F_{k,r}$ -free graph on  $n$  vertices, then  $q(G) \leq q(S_{n, k(r-2)})$ , equality holds if and only if  $G = S_{n, k(r-2)}$ .  
I.e.,

$$ex(n, F_{k,r}) = q(S_{n, k(r-2)}).$$

D. N. Desai, L. Y. Kang, Y. T. Li, Z. Y. Ni, M. Tait, [Spectral extremal graphs for intersecting cliques](#), arXiv: 2108.03587v2.

## Signless Laplacian for flowers

### Theorem 23 (Chen-Liu-Z, 2021+)

Let  $G$  be an  $F_{a_1, \dots, a_k}$ -free graph of order  $n \geq 8t^2 - 12t + 9$  with  $t = \sum_{i=1}^k a_i$ .

- (1) [He-Jin-Z, 2013] If  $k = 1$  and  $a_1 = 1$ , then  $q(G) \leq q(S_{n,t})$  with equality if and only if  $G$  is a complete bipartite graphs  $K_{r,s}$
- (2) [Yuan, 2014] If  $k = 1$ ,  $a_1 \geq 2$ , and  $n \geq 110t^2$ , then  $q(G) \leq q(S_{n,t})$  with equality if and only if  $G = S_{n,t}$ .
- (3) [Zhao-Huang-Guo, 2021] If  $k \geq 2$  and  $a_1 = \dots = a_k = 1$ , then  $q(G) \leq q(S_{n,t})$  with equality if and only if  $G = S_{n,t}$ .
- (4) If  $k \geq 2$  and  $a_1 \geq 2$ , then  $q(G) \leq q(S_{n,t})$  with equality if and only if  $G = S_{n,t}$ .

B. He, Y.-L. Jin, X.-D. Zhang, Sharp bounds for the signless Laplacian spectral radius in terms of clique number, *Linear Algebra Appl.* 438 (2013) 3851–3861.

X. Yuan, Maxima of the  $Q$ -index: forbidden odd cycles, *Linear Algebra Appl.* 458 (2014) 207–216.

Y. Zhao, X. Huang, H. Guo, The signless Laplacian spectral radius of graphs with no intersecting triangles, *Linear Algebra Appl.* 618 (2021) 12–21.

M.-C. Chen, A.-M. Liu, X.-D. Zhang, The signless Laplacian spectral radius of graphs without intersecting odd cycles, arXiv:2108.03895.

### Lemma 27 (Chen-Z, 2019)

Let  $H = \bigcup_{i=1}^k P_{2a_i}$  with  $k \geq 2$ ,  $a_1 \geq \dots \geq a_k \geq 1$ , and  $t = \sum_{i=1}^k a_i$ . If  $\delta(G) \geq t - 1$  and  $G$  is an  $H$ -free connected graph of order  $n \geq 2t$ . then one of the following holds:

- (1)  $G \subseteq S_{n,t-1}$ ;
- (2)  $F = 2P_{2a_1}$  and  $G = L_{r,t-1}$ , where  $n = r(t - 1) + 1$ .

### Lemma 28 (Nikiforov-Yuan, 2014)

Let  $t \geq 2$  and  $n > 5t^2$ . Then

- (1)  $q(S_{n,t}) > n + 2t - 2 - \frac{2(t^2-t)}{n+2t-3} > n + 2t - 3$ .
- (2) If  $G$  is a graph of order  $n$  with  $q(G) \geq q(S_{n,t})$ , then  $e(G) \geq tn - t^2 + 1$ .

M.-Z. Chen, X.-D. Zhang, [Erdos-Gallai stability theorem for linear forests](#), *Discrete Math.* 342 (2019) 904–916.

V. Nikiforov, X. Yuan, [Maxima of the  \$Q\$ -index: graphs without long paths](#), *Electron. J. Linear Algebra* 27 (2014) 504–514.

